**4Unit-3 Laplace Transform**

**1. Motivation:** Laplace transform a very powerful technique is that it replaces operations of calculus by operations of algebra. Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equations. It finds very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics and signal processing. Laplace transforms help in solving complex problems with a very simple approach.

**2. Prerequisite:**

Function, the concept of limit, continuity, ordinary derivative of function, rules and formulae of differentiation and integration of function of one independent variable.

**3. Objective:** The Laplace transform method solves [differential equations](http://mathworld.wolfram.com/OrdinaryDifferentialEquation.html) and corresponding initial and boundary value problems. The Laplace transforms reduce the problem of solving a differential equation to an algebraic problem. It is also useful in problems where the mechanical or electrical driving force has discontinuities, is impulsive or is a complicated periodic function.

The Laplace transform also has the advantage that it solves problems directly, initial and boundary value problems without determining a general solution.

**4. Key Notations:**

1. Laplace transform of a function and  Inverse Laplace transform of a function

**5. Key Definitions:**

**(1)**  **LAPLACE TRANSFORM:** Let be a function defined for all positive values of  , Then  provided the integral exists, is called the Laplace Transform of .

It is denoted as



**(2) INVERSE LAPLACE TRANSFORM:** If then  is called the

Inverse Laplace transform of.

It isdenoted as.

**6. Important Formulae/ Theorems / Properties:**

**LAPLACE TRANSFORM:**

**STANDARD FORMULAE:**

1)  

2)  

3) 

4) 

5)  

6)  

7) 

**7. SAMPLE PROBLEMS:**

**I.Exercise can be solved based on following sample problem.**

**LAPLACE TRANSFORM BY DEFINITION:**

**Ex. Find** the Laplace transform of



**Solution:** By the definition of Laplace transform we have,



**Unsolved Problem**

1)  

2) 

 

 

**HEAVISIDE UNIT STEP FUNCTION**

**LAPLACE TRANSFORM OF HEAVISIDE UNIT STEP FUNCTION:**

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**Ex.** Expressthe function as Heaviside’s unit step functions andfind theirLaplace transform.

**Solution:** By the formulae of Heaviside’s unit step function, we have





**Unsolved Problem**

1) Prove that 

2) Prove that 

3) Evaluate 

4) Evaluate 



6)  



7)  



**LAPLACE TRANSFORM OF DIRAC-DELTA (UNIT IMPULSE) FUNCTIONS**

****

**Ex.** Find the Laplace transform of.

**Solution:** By taking, we have

****

**Unsolved Problem**

1) Prove that 

2) Prove that 

3) Find  

4) Prove that 

5) Prove that 

**LAPLACE TRANSFORM OF PERIODIC FUNCTION**

If f (t) is a periodic function of period T then 

**Ex. Find** the Laplace transform of  and f (t) is periodic with period 2a.

**Solution:** Since f (t) is periodic with period 2a.

****



**Unsolved Problem**

If is a periodic function of period T then 

1) Find Laplace Transform of  

2) Find Laplace Transform of 



3) Find Laplace Transform of



**II Exercise can be solved based on following sample problem.**

**LAPLACE TRANSFORM BY LINEARITY PROPERTY:**



**Ex.** Find the Laplace transform of 

**Solution:** By linearity property, we have

****

**Unsolved Problem**

Evaluate

1)  

2)  

3)  

4)  

5)  

6)  

**CHANGE OF SCALE PROPERTY**:

If  then 

**Ex.** If**,** find.

**Solution:** By change of scale property, we have





**Unsolved Problem**

**If  then **

1) Find  if  

2) Find  

3) 

**FIRST SHIFTING THEOREM**:

If  

**Ex.** Find the Laplace transform of.

**Solution:** We know that















Form (1), (2) and (3), we get



**Unsolved Problem**

**If  **

Evaluate

1)  

2)  

3)  

4)  

5)  

6)  

7)  

8)  

**EFFECT OF MULTIPLICATION BY**:



**Ex.** Find the Laplace transform of.

**Solution:**













**Unsolved Problem**



Evaluate

1)  

2)  

3)  

4)  

5)  

6)  

7)  

8)  

**EFFECT OF DIVISION BY t**

If 

**Ex.** Find 

**Solution:** We know that

****

By effect of division, we have











Integrating by parts





**Unsolved Problem**

If 

Evaluate

1)  

2)  

3)  

4)  

5)  

**LAPLACE TRANSFORM OF DERIVATIVE**



**Ex.** Find and, where 

**Solution:** 









But is an indeterminate form, which can be solved by L’Hospital’s Rule





**Unsolved Problem**

1) 

i) If  

 

 

2) If     
 

**LAPLACE TRANSFORM OF INTEGRAL**

If  

**Ex.**Find the Laplace transform of 

**Solution:**

****

****

****

**Unsolved Problem**

Evaluate

1)  

2)  

3) 

4)  

5)  

6)  

7)  

**EVALUATION OF INTEGRAL USING LAPLACE TRANSFORMS**

**Ex.** Evaluate 

**Solution:** By comparing the given integral  with the Definition of Laplace transform L [f (t)] =  we get,

s=1 and 

Now, 



Now, 

Putting s=1, we get



**Unsolved Problem**

1) Show that 

2) Show that 

4) Show that 

5) Show that 

6) Show that 

7) Show that 

8) If  

**INVERSE LAPLACE TRANSFORM:**

**STANDARD FORMULAE:**

1) 

2) 

3) 

4) 

5) 

6) 

7) 

8) 

**III Exercise can be solved based on following sample problem.**

**INVERSE BY DIRECT FORMULAE**

**Ex.** Find the inverse Laplace transform of 

**Solution:**

****

**Unsolved Problem**

Evaluate

1) 

2) 

3) 

**INVERSE BY FIRST SHIFTING THEOREM**

****

**Ex.** Find the inverse Laplace transform of 

**Solution:**





By First shifting theorem, we have







**Unsolved Problem**

Evaluate

1) 

2) 

3) 

**INVERSE BY PARTIAL FRACTION**

**Ex.** Find the inverse Laplace transform of 

**Solution:**

****

****

Let

And hence





When x=-1, 1=3b; when x=-4, -2=-3a









**Unsolved Problem**

Evaluate

1)  

2) 

4) 

5)  .

7)  

8)  

11)  

12)  

13)  

**INVERSE BY CONVOLUTION THEOREM**

Let 





**Ex.** Find the inverse Laplace transform of 

**Solution:** By convolution theorem, we have







**Unsolved Problem**

Evaluate

1) 

2)  3)

4) 

5)  

6)  

7)  

8)  

**INVERSE BY DIFFERENTIATION OF **

If 

**Ex.** Find the inverse Laplace transform of.

**Solution:** By differentiation of (s), we have



**Unsolved Problem**

Evaluate

1)  

2)  

3)  

4)  

5)  

6)  

**INVERSE LAPLACE TRANSFORM BY HEAVISIDE UNIT STEP FUNCTION:**

****

**Ex.** Find the inverse Laplace transform of .

**Solution:** Here, We know that

,



**Unsolved Problem**

1) 

2)  

3) 

4) 

**IV Exercise can be solved based on following sample problem.**

**APPLICATIONS OF LAPLACE TRANSFORM**

**Ex.** Solve the following equation by using Laplace transform

**.**

**Solution:** Let**.** Taking Laplace transform on both the sides, we get



But



The equation becomes



Let 

Putting 

Putting 

Equating the coefficients of , we get





**Ex.** Solve by using Laplace transform



**Solution:** Let**.** Taking Laplace transform on both the sides, we get



But



The equation becomes



Let 

After simplication, we get



**Unsolved Problem**

1) Solve 

2) Solve  

 

4) Solve  

5) Solve  

6) Solve  

7) Solve  

8) Solve  

9) Solve  

**MULTIPLE CHOICE QUESTIONS**

**Choose the correct alternative from each of the following:**

1. If be a function, , then defined as

(a)  (b) 

(c)  (d) 

2. is

(a)  (b) 

(c)  (d) 

3.  is

(a)  (b)

(c)  (d) -

4. If  , then  known as

(a) First shifting theorem (b) Second shifting theorem

(c) Change of scale property (d) Multiplication theorem

 then  is

(a)  b) 

(c)  (d) 

6. is

(a)  (b) 

(c)  (d) 

7. . If  then  is

 

(c) 

8. If  then L is

(a)  (b) 

 

9. If  be a periodic function with period  then is

(a)  (b) 

(c)  (d) 

10. The Laplace transform of Heavisides’s Unit function  is

(a)  (b) 

(c)  (d) 

11. If  then  is

(a)  (b) 

(c)  (d) 

12.  is

(a)  (b) 

(c)  (d) 

13.  is defined as

(a)  (b) sinat

(c)  (d) cosat

14. is defined as

(a)  (b) coshat

(c)  (d) cosat

15.  is a positive integer defined as

(a)  (b) 

(c)  (d) 

16.  defined as

(a)  (b) 

(c)  (d) 

17. If then L  is

(a)  (b)

(c) (d) -

18. Ifand then  is

(a) (b) 

(c)  (d) 

19.If  is

1. 
2. 
3. 
4. None of these

**ANSWER**

**1 (d) 2 (b) 3 (c) 4 (c) 5 (b) 6 (a) 7 (d) 8 (a)**

**9 (d) 10 (c) 11(b) 12(b) 13(a) 14(a) 15(a) 16(c)**

**17(c) 18(b) 19(c) 20(a)**

**Learning Resources:**

* Higher Engineering Mathematics, Dr. B. S. Grewal, Khanna Publications.
* A textbook of Applied Mathematics, P.N. and J. N.Wartikar,Volume 1 and 2, Pune

Vidyarthi Griha.

* Advanced Engineering Mathematics, Erwin Kreyszing, Wiley Eastern Limited,

8th Ed.